Chapter 9

Assignment 9 Solutions

6.5 Consider the avalanche device of the preceding problem.

a. Plot the S/N vs. M for values of M ranging from 1 to 100.

b. Find the approximate value of M_{opt} and S/N at $M = M_{\text{opt}}$ from your graph.

c. Calculate the values of M_{opt} and S/N at $M = M_{\text{opt}}$ from formulas and compare with your graphical results of the previous question.

Solution: a) (Note: We will use $F(M) = M^x = M^{0.5}$ as in the previous problem.) The signal-to-noise ratio is given by

$$\frac{S}{N} = \frac{(m\mathcal{R}_0 P_0 M)^2/2}{2q\mathcal{R}_0 P_0 B M^{2.5} + 2qi_D B M^{2.5} + 2qI_s B + (4kTB/R_L)}\,. \tag{9.1}$$

We know all of the values but M,

$$< i_s^2 >= \frac{(0.85)(0.583)(1\times 10^{-8})(M))^2}{2} = 1.228\times 10^{-17} M^2 \text{ A}^2\,, \tag{9.2}$$

$$\begin{aligned} < i_N^2 >_1 &= 2q\mathcal{R}_0 P_0 B M^{2.5} \\ &= (2)(1.6\times 10^{-19})(0.583)(1\times 10^{-8})(1\times 10^4) M^{2.5} \\ &= 1.868\times 10^{-23} M^{2.5} \text{ A}^2\,, \end{aligned} \tag{9.3}$$

$$\begin{aligned} < i_N^2 >_2 &= 2qI_D B M^{2.5} \\ &= (2)(1.6\times 10^{-19})(1\times 10^{-9})(1\times 10^4) M^{2.5} \\ &= 3.204\times 10^{-24} M^{2.5} \text{ A}^2\,, \end{aligned} \tag{9.4}$$

$$\begin{aligned} < i_N^2 >_3 &= 2qI_s B = (2)(1.6\times 10^{-19})(1\times 10^{-9})(1\times 10^4) \\ &= 3.204\times 10^{-24} \text{ A}^2\,, \end{aligned} \tag{9.5}$$

and

$$< i_N^2 >_4 \;=\; \frac{4kTB}{R_L} = \frac{(4)(1.38\times10^{-23})(300)(1\times10^4)}{1\times10^4} \;=\; 1.657\times10^{-20}\ \mathrm{A}^2\,. \tag{9.6}$$

So we can write the SNR as

$$\frac{S}{N} = \frac{1.228\times10^{-17}M^2}{1.868\times10^{-23}M^{2.5}+3.204\times10^{-24}M^{2.5}+3.204\times10^{-24}+1.657\times10^{-20}} = \frac{1.228\times10^{-17}M^2}{(2.19\times10^{-23})M^{2.5}+1.656\times10^{-20}}\,.$$

This function is plotted in Fig. 9.1.

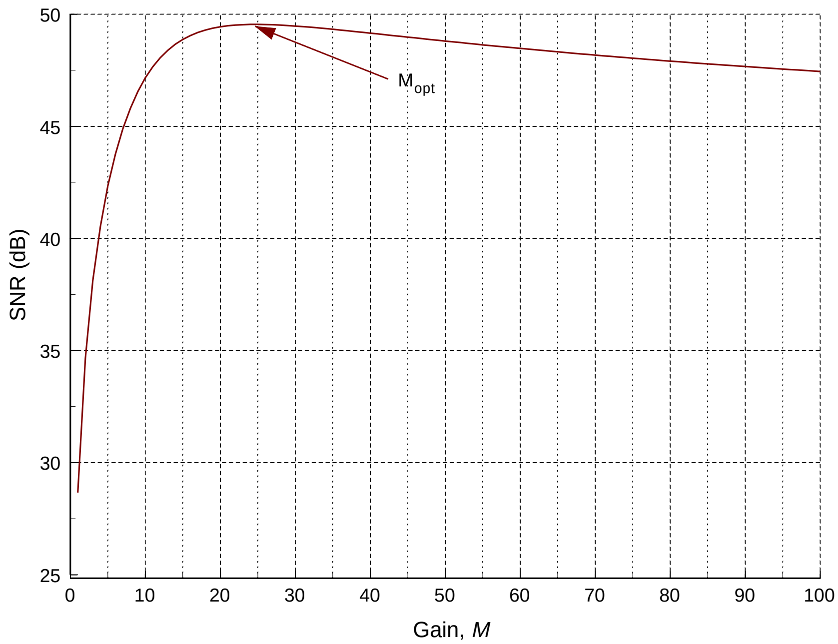

Figure 9.1: Plot of S/N vs. M for Prob. 6.4a.

b) From the plot we see that $M_{\mathrm{opt}} \approx 25$ and that the maximum signal-to-noise ratio is about 49.5 dB.

The *computed* value of M_{opt} is found from

$$M_{\mathrm{opt}}^{x+2} \approx \frac{2qI_s + (4kT/R_L)}{xq(\mathcal{R}_0P_0 + I_D} \tag{9.7}$$

$$
\begin{aligned}
M_{\text{opt}}^{2.5} &\approx \frac{(2)(1.6\times10^{-19})(1\times10^{-9}) + \dfrac{(4)(1.38\times10^{-23})(300)}{1\times10^{4}}}{(0.5)(1.6\times10^{-19})((0.583)(1\times10^{-8})+1\times10^{-9}} \\
&\approx 3.03\times10^{3} \\
M_{\text{opt}} &\approx (3.03\times10^{3})^{1/2.5} = 24.7\,.
\end{aligned}
$$

The signal-to-noise ratio at the optimum value of M is

$$
\begin{aligned}
\frac{S}{N} &= \frac{1.228\times10^{-17}M_{\text{opt}}^{2}}{(2.19\times10^{-23})M_{\text{opt}}^{2.5}+1.656\times10^{-20}} \qquad (9.8) \\
&= \frac{1.228\times10^{-17}(24.7)^{2}}{(2.19\times10^{-23})(24.7)^{2.5}+1.656\times10^{-20}} \\
&= 9.03\times10^{4} \Rightarrow 49.55\ \text{dB}\,.
\end{aligned}
$$

These calculated results compare favorably with the graphical results.

6.6. Consider a silicon avalanche photodiode with parameters as given below operating in a link with no intersymbol interference present.

$F(M) = M^{0.4}$
Responsivity (at $M = 1$) = 0.3 A/W
Surface dark current = 1 μA
Temperature = 300 K
R_L = 1 kΩ
Bulk dark current = 1 nA
Bandwidth of receiver = 10 MHz

a. Calculate the dc optical power that must be incident on the detector to make the optimum gain of this amplifier have a value of 80.

b. With a value of gain of 80 calculate the ratio (in dB) of the mean-square noise current due to the shot noise caused by the bulk dark current to the mean-square noise current due to the thermal noise.

Solution: We are given that $F(M) = M^{0.4}$, $\mathcal{R}_0 = 0.3$, $I_s = 1\times10^{-6}$ A, $T = 300$K, $R_L = 1\times10^{3}$, $I_D = 1\times10^{-9}$, and $B = 1\times10^{6}$.

a. We are given that $M_{\text{opt}} = 80$ and find the power P from

$$
\begin{aligned}
M_{\text{opt}}^{x+2} &\approx \frac{2qI_s + (4kT/R_L)}{xq(\mathcal{R}_0P_0 + I_D)} \qquad (9.9) \\
(80)^{2.4} &\approx \frac{(2)(1.6\times10^{-19})(1\times10^{-6}) + \dfrac{(4)(1.38\times10^{-23})(300)}{1\times10^{3}}}{(0.4)(1.6\times10^{-19})(0.3P+1\times10^{-9})}\,, \qquad (9.10)
\end{aligned}
$$

so

$$
\begin{aligned}
0.3P + 1\times 10^{-9} &= \frac{1.692\times 10^{-23}}{(0.4)(1.6\times 10^{-19})(80)^{2.4}} = 7.159\times 10^{-9} \qquad (9.11)\\
0.3P &= 6.159\times 10^{-9}\\
P &= 2.05\times 10^{-8}\ \mathrm{W} = 20.5\ \mathrm{nW}\,.
\end{aligned}
$$

b. The ratio of the shot noise due to the dark current ($< i_N^2 >_1$) to the noise due to thermal noise ($< i_N^2 >_2$) is

$$
\begin{aligned}
\frac{< i_N^2 >_1}{< i_N^2 >_2} &= \frac{2qI_D BM^2F(M)}{4kTB/R_L} \qquad (9.12)\\
&= \frac{(2)(1.6\times 10^{-19})(1\times 10^{-9})(80)^{2.4}}{[(4)(1.38\times 10^{-23})(300)/(1\times 10^3)]}\\
&= 0.710 \Rightarrow -1.466\ \mathrm{dB}\,.
\end{aligned}
$$

6.7. Consider a silicon photodiode operating at 850 nm ($\alpha = 10^3$ cm^{-1}).

a. Calculate the area of the device if the capacitance is to be kept equal or less than 2 pF. The relative permittivity of silicon is 11.7.

b. Calculate the maximum bandwidth of the detector when operating into 50 Ω load.

<u>Solution:</u> a) We estimate the width w of the depletion region as

$$w = \frac{2}{\alpha} = \frac{2}{1\times 10^5} = 2\times 10^{-5}\ \mathrm{m}\,. \qquad (9.13)$$

The capacitance is required to obey $C \le 2\times 10^{-12}$, so

$$C_{\max} = 2\times 10^{-12} \qquad (9.14)$$

$$\frac{\epsilon_r\epsilon_0 A_{\max}}{w} = 2\times 10^{-12} \qquad (9.15)$$

$$A_{\max} = \frac{(2\times 10^{-12})w}{\epsilon_r\epsilon_0} = \frac{(2\times 10^{-12})(2\times 10^{-5})}{(11.7)(8.85\times 10^{-12})} = 3.86\times 10^{-3}\ \mathrm{m}^2\,. \qquad (9.16)$$

We can find the diameter of a circular detector from

$$A_{\max} = \frac{\pi d_{\max}^2}{4} \qquad (9.17)$$

$$d_{\max} = \sqrt{\frac{4A_{\max}}{\pi}} = \sqrt{\frac{(4)(3.86\times 10^{-3})}{\pi}} = 701\times 10^{-4}\mathrm{m} = 0.701\ \mu\mathrm{m}\,. \qquad (9.18)$$

The detector is very small, indeed!!

b) The maximum frequency that this detector can respond to when used with a 50 Ω load is

$$f_{\max} = \frac{1}{2\pi R_L C} = \frac{1}{2\pi(50)(2\times 10^{-12})} = 1.591\times 10^9 \text{ Hz} = 1.591 \text{ GHz}\,. \tag{9.19}$$

6.8. Consider a pin diode with the following properties at 920 nm—

- Responsivity = 0.5 A/W
- Dark current = 1.0 nA
- Surface dark current is negligible
- Operating temperature = 300 K.

This diode is irradiated with a constant 80 nW of optical power (at 920 nm). Find the signal-to-noise ratio *(in dB)* of the detector if it is operated into an equivalent load of 10 KΩ and a (noisefree) preamp with a bandwidth of 1 MHz.

Solution: The mean-square signal is found from

$$< i_s^2 >= (\mathcal{R}_0 P)^2 = \Big((0.5)(8\times 10^{-8})\Big)^2 \; = 1.6\times 10^{-15} \text{ A}^2\,. \tag{9.20}$$

The mean-square noise current is found as

$$\begin{aligned} < i_N^2 > &= 2q\left(\mathcal{R}_0 P + I_D\right)B + 2qI_{\text{surface}}B + \frac{4kTB}{R_L} \qquad (9.21)\\ &= 2q\Big((0.5)(8\times 10^{8}) + 1\times 10^{-9}\Big)(1\times 10^6) + 0 \\ &\quad + \frac{4(1.38\times 10^{-23})(300)(1\times 10^6)}{1\times 10^4} \\ &= 1.669\times 10^{-18} \text{ A}^2\,. \end{aligned}$$

The signal-to-noise ratio is

$$\frac{S}{N} = \frac{1.6\times 10^{-15}}{1.669\times 10^{-18}} = 959 \Rightarrow 19.8 \text{ dB}\,. \tag{9.22}$$

6.9. Show that the relation between the Q-parameter and S/N is

$$Q = \frac{1}{2}\sqrt{\frac{S}{N}}\,. \tag{9.23}$$

Solution: Equation 5.171 can be written as

$$\mathrm{BER} = 0.5\,\mathrm{erf}\left(\frac{Q}{\sqrt{2}}\right) . \tag{9.24}$$

Equation 5.152 is

$$\mathrm{BER} = 0.5\,\mathrm{erf}\left(\frac{\sqrt{\mathrm{SNR}}}{2\sqrt{2}}\right) , \tag{9.25}$$

so,

$$Q = \frac{\sqrt{\mathrm{SNR}}}{2} = \frac{1}{2}\sqrt{\frac{S}{N}} . \tag{9.26}$$